# *Com$^2$MaC Mini-Workshop on* **Two-face embeddings of graphs and applications**

*Edited by*

Michel-Marie Deza

and

Jin Ho Kwak

Supported by Com$^2$MaC-KOSEF, Korea.

January 5 - January 14, 2004

# Contents

# Com$^2$MaC Mini-Workshop on Two-face embeddings of graphs and applications

**ORGANIZERS**

Michel-Marie DEZA (Ecole Normale Superieure, France)
Jin Ho Kwak (POSTECH, Korea)

**INVITED SPEAKERS**

Michel-Marie DEZA (Ecole Normale Superieure, France)
Mathieu Dutour (Hebrew University, Isreal)
Yan Quan Feng (Northern Jiaotong University, China)
P.W. Fowler (Exeter University, UK)
Dal Young Jeong (Soong Sil University, Korea)
Jacobus H. Koolen (KAIST, Korea)
Jin Ho Kwak (POSTECH, Korea)
Young Soo Kwon (POSTECH, Korea)
Jaeun Lee (Yeungnam University, Korea)
Alexander D. Mednykh (Novosibirsk University., Russia)
Atsuhiro Nakamoto (Yokohama University, Japan)
Roman Nedela (Matej Bel University, Slovakia)
Seiya Negami (Yokohama University, Japan)
Elena Pantelleeva (Moscow Pedagogical University, Russia)
Iwao Sato (Oyama National College of Technology, Japan)
YoungHee Shin (Yeungnam University, Korea)

**Sponsors**
**Ministry of Science and Technology of Korea**
**Korea Science and Engineering Foundation**

*****

We plan to make a proceeding of the workshop. Detailed information will be provided later.

# Part I

# Program
# January 13, 2004

*Location: Room 402 of Engineering Building 5 in POSTECH*

## Monday, January 5, 2004

14:00 - 15:00 *Orientation*

## Tuesday, January 6, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Jin Ho Kwak** | Introduction to Tessellation and (Semi-) Regular Maps |
| 14:00 - 15:00 | **P. W. Fowler** | Fullerenes: a Favourable Case for Interaction between Mathematics and Chemistry (I) |
| 15:30 - 16:30 | **P. W. Fowler** | Fullerenes: a Favourable Case for Interaction between Mathematics and Chemistry (II) |

## Wednesday, January 7, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Michel-Marie Deza** | Zigzag Structute of Simple Polyhedra |
| 14:00 - 15:00 | **Mathieu Dutour** | Goldberg-Coxeter construction for 3- or 4-valent maps |
| 15:30 - 16:30 | **Mathieu Dutour** | Polycycles and their boundaries |

## Thursday, January 8, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Dianjun Wang** | Semi-Symmetrical Maps |
| 14:00 - 15:00 | **Jacobus H. Koolen** | Teutoburgan Split Systems |
| 15:30 - 16:30 | **Young-hee Shin** | Balanced regular coverings of a signed graph and regular branched orientable surface coverings over a non-orientable surface |

## Friday, January 9, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Jaeun Lee** | A construction method of network mappings |
| 14:00 - 15:00 | **Elena Pantelleeva** | Integer Points in Some Domains |
| 15:30 - 16:30 | **Iwao Sato** | Zeta functions of regular coverings of graphs |
| 18:00- | | *Banquet* |

## Saturday, January 10, 2004

*Gyeongju Tour*

## Monday, January 12, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Seiya Negami** | Diagonal flips in graphs on closed surfaces with triangular and quadrangular faces |
| 14:00 - 15:00 | **Atsuhiro Nakamoto** | Generating 3-connected quadrangulations on surfaces |
| 15:30 - 16:30 | **Young Soo Kwon** | Regular Maps and Cayley Maps |

## Tuesday, January 13, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Roman Nedela** | Chirality group and chirality index of maps and hypermaps |
| 14:00 - 15:00 | **Dal Young Jeong** | TBA |
| 15:30 - 16:30 | **Yan Quan Feng** | Classifying $S$-Regular Cubic Graphs of Order $2P^2$ |

## Wednesday, January 14, 2004

| Time | Speaker | Lecture Title |
|---|---|---|
| 09:30 - 10:50 | **Alexander D. Mednykh** | Geometry of Polyhedra and Knot Theory |
| 14:00 - 15:00 | **Sejeong Bang** | Improving diameter bounds for distance-regular graphs |
| 15:30 - 16:30 | **YoungBin Choe** | Polynomials with the half-plane property and the support theorems |

# Part II

# Abstracts

# Improving diameter bounds for distance-regular graphs

S. Bang, A. Hiraki, Jack Koolen

sjbang3@postech.ac.kr

*Combinatorial and Computational Mathematics Center*
*POSTECH*
*Pohang, 790-784, Korea*

In this paper we study for given $c$, the total number of $i$'s such that $c_i = c$ and we call this number $\eta_c$ for a distance-regular graph. We show that all $i$ with $c_i = c$ satisfy $i > \eta_c$. We also give improvements on diameter bounds by Hiraki and Koolen, and Pyber, respectively, by applying this inequality.

# Chirality group and chirality index of maps and hypermaps

A. Breda, G. Jones, R. Nedela and M. Skoviera

nedela@savbb.sk

*Department of Mathematics*
*Matej Bel University*
*975 49 Banská Bystrica, Slovakia*

A geometric object is called reflexible or chiral as it is or is not isomorphic to its mirror image. We introduce the chirality group and the chirality index of an arbitrary regular hypermap, as algebraic and numerical measures of its lack of mirror symmetry. We establish some general properties of chirality groups, and show that certain classes of groups may or may not arise in this way.

We show how to modify the above measures of chirality to maps and (in particular) for two-face maps.

# Polynomials with the half-plane property and the support theorems

YoungBin Choe

ybchoe@postech.ac.kr

*Combinatorial and Computational Mathematics Center*
*POSTECH*
*Pohang, 790-784, Korea*

A polynomial $P(x)$ in $n$ complex variables is said to have the half-plane property if $P(x) \neq 0$ whenever all the variables have positive real parts. The generating polynomial for the set of all spanning trees of a graph $G$ is one example. Motivated by the fact that the edge set of each spanning tree of $G$ is a basis of the graphic matroid induced by $G$, we can show that the support of any homogeneous multiaffine polynomial with the half-plane property constitutes the set of all the bases of some matroid. In this paper we show that by relaxing the multilinearity condition, when the polynomial has definite parity with the half-plane property, the support constitutes a jump system which is a generalization of matroids. Open problems and a few directions for further research will also be discussed.

# Zigzag Structute of Simple Polyhedra

Michel Deza

deza@liga.ens.fr

*ENS/CNRS, Paris and ISM, Tokyo*

A zigzag in a plane graph is a circuit of edges, such that any two, but no three consecutive edges belong to the same face. A railroad in a plane graph is a circuit of hexagonal faces, such that any hexagon is adjacent to its neighbors on opposite edges. We consider the zigzag and railroad structure of simple two-faced polyhedra and some generalizations of those notions.

# Goldberg-Coxeter construction for - or -valent plane graphs

Mathieu Dutour

dutour@liga.ens.fr

*ENS/CNRS, Paris and Hebrew University, Jerusalem*

Michel Deza

deza@liga.ens.fr

*ENS/CNRS, Paris and ISM, Tokyo*

We consider Goldberg-Coxeter construction $GC_{k,l}(G_0)$, which produces a plane graph from any 3- or 4-valent plane graph and integer parameters $k, l$.

A zigzag in a plane graph is a circuit of edges, such that any two, but no three, consecutive edges belong to the same face; a central circuits in a 4-valent plane graph G is a circuit of edges, such that any two consecutive edges do not belong to the same face.

We study zigzag (or central circuit) structure of $GC_{k,l}(G_0)$ using algebraic formalism of the moving group, the $(k, l)$-product and a finite index subgroup of $SL_2(\mathbb{Z})$, whose elements preserve above structure.

I will then explain the possible generalizations of this construction.

# Polycycles and their boundaries

Mathieu Dutour, Michel Deza and Misha Shtogrin

dutour@liga.ens.fr

*ENS/CNRS, Paris and Hebrew University, Jerusalem*

TBA

# Integer Points in Some Domains

Elena Deza

pantel@mccme.ru

*Department of Mathematics*
*Moscow State Pedagogical University*
*14, Krasnoprudnya, Moscow, Russia*

We consider some problems, connected with the calculating of the number of integer points in some special domains. Classic examples of such problems are the Gauss Circle problem about the calculating the amount of integer points in a circle and the Dirichlet divisor problem about the calculating the amount the positive integer points under a hyperbola.

If we consider the hyperbola $x_1 \cdot x_2 = x$, then the number of integer points under it is

$D_2 = \sum_{x_1 x_2 \le x} 1 = \sum_{n \le x} \sum_{x_1 x_2 = n} 1 = \sum_{n \le x} \sum_{d/n} 1 = \sum_{n \le x} \tau(n)$,

and we come to the problem of the research of asymptotic behavior of the divisor function $\tau(n)$ on the average.

Consider now the function $\tau_k(n)$ as the number of representations of the positive integer number $n$ as a product of $k$ positive integer factors. Hence, $\tau_2(n) = \tau(n)$ and $\tau_k(n)$ is a generalization of the function of divisors.

We obtain new results about mean value of $\tau_k(n)$ for large $k$:

$D_k(x) = \sum_{n_1 \cdot \ldots \cdot n_k \le x} 1 = xP_k(\log x) + \Theta x^{1-\frac{1}{13k^{-2/3}}} (C \log x)^k$,

where $P_k$ is a polynomial of degree $k-1$, $\Theta$ is a complex valued number, $|\Theta| \le 1$, $2 \le k \le \log x$, $C > 0$ - absolute constant.

We obtain similar result for a generalization of the function $D_k(x)$:

$C_k(x) = \sum_{n_1 \cdot \ldots \cdot n_k \le x} \chi_1(n_1) \cdot \ldots \cdot \chi_k(n_k) = xP_m(\log x) + \Theta x^{1-\frac{1}{13k^{-2/3}}} (C \log x)^k$, where $\chi_1, ..., \chi_k$ are Dirichlet characters modulo $D_1, ..., D_k$, $\max(D_1, ..., D_k) \le \log^2 x$, $m$ is the number of principal characters from $\chi_1, ..., \chi_k$, $P_m$ is a polynomial of degree $m-1$, $\Theta$ is a complex valued number, $|\Theta| \le 1$, $2 \le k \le \log x$, $C > 0$ - absolute constant.

At last, we obtain new results for mean values of two arithmetic functions, similar of the divisor function:

$\sum_{n \le x} \tau_{k_1}(n) \cdot \ldots \cdot \tau_{k_l}(n) = xP_m(\log x) + \Theta x^{1-\frac{1}{13m^{-2/3}}} (C \log x)^m$,

where $l \ge 1$,$k_1, ..., k_l \ge 2$, $m = k_1 \cdot \ldots \cdot k_l$, $P_m$ is a polynomial of degree $m-1$, $\Theta$ is a complex valued number, $|\Theta| \le 1$, $m \le \log x$, $C > 0$ - absolute constant;

$\sum_{n \le x} \tau_k(n^2) = xP_m(\log x) + \Theta x^{1-\frac{1}{13m^{-2/3}}} (C \log x)^m$, where $m = k(k+1)/2$, $P_m$ is a polynomial of degree $m-1$, $\Theta$ is a complex valued number, $|\Theta| \le 1$, $m \le \log x$, $C > 0$ - absolute constant.

**References**

1. Dirichlet P. Uber die Bestimmung der mittleren Werte in der Zahlentheorie. Berlin: Abh. Acad. Wiss. 1849. Werke 2. S. 49 - 66.

2. Panteleeva E. About mean values of some arithmetic functions. Math Notes. 1994. V. 55 - 2. P. 109 - 117.

# Classifying $S$-Regular Cubic Graphs Of Order $2P^2$

Yan-Quan Feng

*Department of Mathematics*
*Beijing Jiaotong University*
*Beijing, P.R. China*

An *s-arc* in a graph is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_{s-1}, v_s)$ of vertices of the graph such that any two consecutive vertices are adjacent and any three consecutive vertices are distinct. A graph $X$ is said to be *s-arc-transitive* or *s-regular* if its full automorphism group acts transitively or regularly on the set of $s$-arcs in $X$, respectively. Tutte showed that every finite connected 1-arc transitive graph is $s$-regular for some $s \geq 1$, and this $s$ should be at most five. In this report, we discuss the classification of $s$-regular cubic graphs of order $2p^2$ for each prime $p$ and each $1 \leq s \leq 5$.

# Fullerenes: a favourable case for interaction between mathematics and chemistry

Patrick Fowler

P.W.Fowler@exeter.ac.uk

*Dept of Chemistry University of Exeter*
*EXETER EX4 4QD UK*

Fullerene chemistry and physics is an area where two-way interaction between chemistry and mathematics is particularly strong. Some applications of graphs and their embeddings will be discussed for these bifaced polyhedra, with emphasis on the variety of present results in chemistry and the future directions for joint chemical-mathematical exploration.

Systematics of fullerene physics and chemistry can be developed using a mixture of symmetry, combinatoric and graph theoretical arguments. Isomer counts, magic numbers in electronic structure, energetics and patterns of addition chemistry can all be understood with symmetry/graph based models. Adjacency matrices and their eigenvalues partially characterise stability and reactivity of the unsaturated fullerene carbon frameworks, Other matrices are less well exploited in chemistry; eigenvectors of bond-polarisability matrices characterise the degree of distortivity of the framework; distance matrices, independence numbers and codes all model the addition chemistry of fullerenes. Qualitative relations between distance and adjacency spectra are observed, and constructions for specific ‘leapfrog’ fullerenes suggest chemically relevant conjectures. Embeddings of fullerene and related graphs on 2D manifolds lead to electronic-structure rules for toroidal and more exotic fullerene anaogues and to some chemically useful relations for mobius molecules.

## References


1. P.W. Fowler and D.E. Manolopoulos, An atlas of fullerenes, OUP (1995)

2. P.W. Fowler ,P. Hansen, K.M. Rogers and S. Fajtlowicz, J. Chem. Soc. Perkin 2 (1998) 1531-1533 C60Br24 as a chemical illustration of graph theoretical independence; P.W. Fowler, B. de La Vaissiere and M. Deza, J. Mol. Graphics and Modelling 19 (2001) 199-204 Addition patterns, codes and contact graphs for fullerene derivatives; B. de La Vaissiere, P.W. Fowler and M. Deza J. Chem. Inf. Comp. Sci., 41 (2001) 376-386 Codes in Platonic, Archimedean, Catalan and related polyhedra: a model for maximum addition patterns in chemical cages

3. P.W. Fowler, G. Caporossi and P. Hansen, J. Phys. Chem. A 105 (2001) 6232-6242 Distance Matrices, Wiener Indices and Related Invariants of Fullerenes

4. M. Deza, P.W. Fowler, A. Rassat and K.M. Rogers, J. Chem. Inf. Comp. Sci. 40 (2000) 550-558 Fullerenes as tilings of surfaces

5. P.W. Fowler, Phys. Chem. Chem. Phys. 4 (2002) 2878-2883 Huckel spectra of Mobius pi systems

# Teutoburgan Split Systems

J. H. Koolen

jhk@amath.kaist.ac.kr

*KAIST, Korea*

In 1964, J. Isbell introduced the concept of the injective hull of a metric which, in case the metric is finite (i.e. it is defined on a finite set), constitutes a polytopal complex with interesting combinatorial and geometrical properties. In general, the injective hull or, as we shall call it, the tight-span of a finite metric is a complicated object to understand, but for certain classes of metrics, much can be said. In this talk we will consider the tight-span of a totally split-decomposable metric, a polytopal complex that has been studied by Dress, Huber and Moulton. In particular, amongst other things, we will show that every cell in the tight-span of a totally split-decomposable metric is a zonotope that is polytope isomorphic to either a hypercube or the rhombic dodecahedron.

This is joint work with K. Huber and V. Moulton, both from Uppsala, Sweden.

# A construction method of network mappings

Jin Ho Kwak

jinkwak@postech.ac.kr

*Combinatorial and Computational Mathematics Center*
*POSTECH, Pohang 790-784, Korea*

Jaeun Lee

julee@yu.ac.kr

*Depatment of Mathematics*
*Yeungnam University, Kyongsan 712-749, Korea*

Most standard network models, like Hypercube. Butterfly, Wrapped-butterfly, Cube-connected-cycles, etc., are described by using bit binary sting, and have been examined mostly by using algebraic tools like number theory or group theory. In this paper, we introduce a new topological method to discribe them as covering networks over simple base networks by using global or local symmetries of networks. By using this new description, we obtain a method to construct network mappings between large interconnection networks when a mapping is given between their base networks, and a formula to measure the efficiency of the constructed mapping.

# Balanced regular coverings of a signed graph and regular branched orientable surface coverings over a non-orientable surface

Jin Ho Kwak

jinkwak@postech.ac.kr

*Combinatorial and Computational Mathematics Center*
*POSTECH, Pohang 790-784, Korea*

Jaeun Lee and Young-hee Shin

jina_2@hanmail.net, julee@yu.ac.kr

*Depatment of Mathematics*
*Yeungnam University, Kyongsan 712-749, Korea*

The isomorphism classes of several types of graph coverings of a graph have been enumerated by many authors. Kwak and Lee enumerated the isomorphism classes of $n$-fold graph coverings of a graph $G$. Recently, Archdeacon, et al. characterized a bipartite covering of $G$ and enumerated the isomorphism classes of regular $2p$-fold bipartite coverings of a non-bipartite graph. As a continuation of their study, we enumerate the isomorphism classes of regular balanced coverings of a signed graph and those of regular bipartite coverings of a graph. As an application of the result, we also enumerate the isomorphism classes of regular branched orientable surface coverings of a non-orientable surface having a finite abelian covering transformation group. It gives a partial answer for the question raised by Liskovets.

# Semi-Symmetrical Maps

Jin Ho Kwak

jinkwak@postech.ac.kr

*Combinatorial and Computational Mathematics Center*
*POSTECH, Pohang 790-784, Korea*

Dianjun Wang

djwang@math.tsinghua.edu.cn

*Department of Mathematical Sciences*
*Tsinghua University*
*Beijing 100084, China*

Let $M$ be a map with underlying graph $X = (V, E)$ and $F$ be the set of faces of map $M$. The map $M$ is said to be semi-symmetrical if $\mathrm{Aut}(M)$ acts transitively on $V$ and has two orbits on $F$. In this talk we present some observations, examples and open problems about semi-symmetrical maps.

# Geometry of Polyhedra and Knot Theory

Alexander D. Mednykh

mednykh@math.nsc.ru

*Sobolev Institute of Mathematics*
*Novosibirsk, Russia*

In the first part of lecture we consider polyhedra in the Euclidean, hyperbolic and hyperbolic spaces. The aim is to find the volume of polyhedra starting from its combinatorial type and prescribe lengths of its edges. In the Euclidean three space this problem was recently solved by I. H. Sabitov. It was shown that the volume is a root of an algebraic equation defined by combinatorial type of polyhedra whose coefficients are integer polynomials in lengths of edges. We discuss the similar problem in hyperbolic and spherical spaces. In particular, the volume formulae for hyperbolic and spherical tetrahedra will be obtained. The second part is devoted to construction of geometrical structures on knots and link complement in the three dimensional sphere. The results of volume calculation for polyhedra have been used to find Euclidean, hyperbolic and hyperbolic volumes for knots and links.

# Generating 3-connected quadrangulations on surfaces

Atsuhiro Nakamoto

nakamoto@edhs.ynu.ac.jp

*Department of Mathematics, Faculty of Education and Human Sciences*
*Yokohama National University, Japan*

Let $G$ be a quadrangulation on a closed surface $F^2$.

- $G$ is *irreducible* if any face contraction breaks the simpleness of $G$,
- $G$ is $\mathcal{D}_3$-*irreducible* if $G$ has minimum degree at least 3 and any face contraction and any 4-cycle removal breaks the simpleness or the minimum degree at least 3 of the graph,
- $G$ is $\mathcal{K}_3$-*irreducible* if $G$ is 3-connected and any face contraction and any 4-cycle removal breaks the simpleness or the 3-connectedness of the graph,
- $G$ is $\mathcal{S}_4$-*irreducible* if $G$ has no separating 4-cycle and any face contraction breaks the simpleness or it creats a separating 4-cycle.

Except the sphere and the projective plane, the irreducibility and the $\mathcal{D}_3$-irreducibility of quadrangulations are equivalent. In this paper, we shall prove that for all surfaces, the $\mathcal{D}_3$-irreducibility and the $\mathcal{K}_3$-irreducibility of quadrangulations are equivalent. We also prove that for the sphere, the porjective plane and the torus, the $\mathcal{D}_3$-irreducibility and the $\mathcal{S}_4$-irreducibility of quadrangulations are equivalent, but this does not hold for surfaces of high genera.

# Diagonal flips in graphs on closed surfaces with triangular and quadrangular faces

Seiya Negami

negami@edhs.ynu.ac.jp

*Department of Mathematics*
*Faculty of Education and Human Sciences*
*Yokohama National University*
*79-2 Tokiwadai, Hodogaya-Ku, Yokohama 240-8501, Japan*

In 1936, Wagner proved that any two triangulations on the sphere with the same number of vertices can be transformed into each other by repeating a local deformation, called a *diagonal flip*, which switches a diagonal in a quadrilateral to another. This results has been generalized to a similar theorem for general surfaces by Negami. His theorem states that given a closed surface $F^2$, any two triangulations on $F^2$ can be transformed into each other, up to homeomorphism, by a sequence of diagonal flips if they have the same and sufficiently large number of vertices.

On the other hand, any two quadrangulations on the sphere also can be transformed into each other by two kinds of local deformations called a *diagonal side* and a *diagonal rotation*. This follows immediately from Nakamoto's general theorem on quadrangulations on closed surfaces. His theorem has the same style as Negami's theorem, but involves an algebraic invariant called a *cycle parity* which controls the equivalence over quadrangulations under those deformations.

A natural question arises; what can we say if triangular and quadrangular faces are mixed? We shall give a positive answer to this question as one expects, defining "diagonal flips" suitably for such graphs on closed surfaces. In particular, we can prove that two graphs embedded on the sphere with the same number of triangular and quadrangular faces can be transformed into each other by diagonal flips, up to homeomorphism.

# Zeta functions of regular coverings of graphs

Iwao Sato

isato@oyama-ct.ac.jp

*Department of Mathematics*
*Oyama National College of Technology, Japan*

TBA

# Tour to Gyeong-Ju on Saturday, Jan. 10, 2004

The tour to Gyeong-Ju is planned on Saturday, Jan. 10, 2004. The people who wish to join the trip will meet at some place to be announced.

**Gyeong-Ju is...**
Gyeong-Ju contains some of the most precious treasures from the Korea's 5000-year history. A historic city with a population of 291,000 as of January 2001, Gyeong-Ju is located 370 kilometers southeast of Seoul on an area of 1,323 square kilometers.

**History**
Gyeong-Ju is known to have been the capital of Silla when Park Hyeokgeose founded the nation in 57 B.C. A birthplace of Silla, it had been the political and cultural center of the kingdom for 990 years since its establishment. The outstanding cultural assets remaining across the city such as Buddhist temples, gold crowns and sculptures still mesmerize visitors. Called by a number of names including Seorabeol and Geumseong in earlier days, it began to be called by the today's name, Gyeongju, in 935 during the Goryeo Dynasty. Administratively, the name was changed to Gyeong-Ju-bu in 1413, then Gyeong-Ju-gun in 1895 during the Joseon Dynasty. Gyeong-Ju became a city in 1995.

# Index